Neutrino masses from operator mixing

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We show that in theories that reduce, at the Fermi scale, to an extension of the standard model with two doublets, there can be additional dimension five operators giving rise to neutrino masses. In particular there exists a singlet operator which cannot generate neutrino masses at the tree level, but generates them through operator mixing. Under the assumption that only this operator appears at the tree level we calculate the neutrino mass matrix. It has the Zee mass matrix structure and leads naturally to bimaximal mixing. However, the maximal mixing prediction for solar neutrinos is very sharp even when higher order corrections are considered. To allow for deviations from maximal mixing a fine-tuning is needed in the neutrino mass matrix parameters. This fine-tuning relates the departure from maximal mixing in solar neutrino oscillations with the neutrinoless double beta decay rate.

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The simplest model for neutrino masses is based on the seesaw mechanism [1–3]. In the seesaw mechanism the standard model (SM) is enlarged with singlet right-handed neutrinos. Then, a Dirac mass term M_D mixing left-handed and right-handed neutrinos is possible. In addition, since right-handed neutrinos do not carry any gauge charge, they can have a Majorana mass M_R without compromising the gauge symmetry. If the right-handed Majorana mass term is very large, as expected for a singlet mass term, very light Majorana neutrino masses for left-handed neutrinos are obtained through the diagonalization of the full mass matrix of neutral fermions, $m_\nu = M_D^2/M_R$, thus justifying the small size of the neutrino masses. Since the Dirac mass term is proportional to the standard Higgs vacuum expectation value, this mechanism provides masses which are $m_\nu \approx \Lambda_F^2/\Lambda$, Λ_F being the Fermi scale and Λ the lepton number breaking scale. This type of behavior is much more general and, in fact, many neutrino mass models can be cast into this form. This can be understood in the following way: if the SM is just the low-energy effective manifestation of some underlying theory, the effects of new physics can be represented by a series of gauge-invariant operators containing the SM fields with higher dimension operators suppressed by powers of the scale of new physics [4–6]. At low energies, the most relevant operators will be those with the lowest dimension, namely, dimension five operators. One can easily see that there is only one gauge-invariant operator of dimension five one can build with the field content of the standard model [5]

$$\mathcal{L}_{\text{seesaw}} = -\frac{1}{4}\frac{1}{\Lambda}(\bar{\tilde{l}} F \vec{\tau} l)(\tilde{\varphi}^\dagger \vec{\tau} \varphi), \tag{1}$$

where l is the standard left-handed doublet of leptons, $\tilde{l} = i\tau_2 l^c$, $l^c = C\overline{l}^T$ (C is the charge conjugation operator), φ is the Higgs doublet and $\tilde{\varphi} = i\tau_2\varphi^*$, $\vec{\tau}$ are the Pauli matrices in $SU(2)$ space, F is a complex symmetric matrix in flavor space [$SU(2)$ and flavor indices have been suppressed] and Λ is a scale related to the scale of new physics. It is clear that this Lagrangian does not conserve generational lepton numbers, but in addition it does not conserve the total lepton number, which is violated in two units. In the SM, lepton number is conserved because the requirement of renormalizability and the small particle content (no right-handed neutrinos, no triplet scalars, etc.) but as long as the spectrum is enlarged there is no strong reason for lepton number conservation. Operator (1) makes this statement explicit. Therefore, this operator will be generated in any extension of the SM that does not conserve lepton number.

When the Higgs scalar develops a vacuum expectation value (VEV), operator (1) will give rise to a neutrino Majorana mass matrix given by

$$M_\nu = F\frac{v^2}{\Lambda}, \tag{2}$$

with $v = \langle\varphi^{(0)}\rangle = 174$ GeV, the SM Higgs VEV. If we take the largest eigenvalue of F to be of order 1 and use the laboratory bound on the τ-neutrino mass, $m_{\nu_\tau} < 18$ MeV, we find that $\Lambda > 10^6$ GeV. Should one take the value suggested by atmospheric neutrino data, $m_{\nu_\tau} \approx 0.06$ eV, one would obtain $\Lambda \approx 5\times 10^{14}$ GeV, a scale which is close to the unification scale; physics at very high-energy scales could affect low-energy physics in a measurable way through neutrino masses. However the relationship between Λ and masses of new particles can be quite different from the naive expectations. For instance, it seems that the Lagrangian of Eq. (1) is generated by the exchange, among the leptons and the Higgses, of a scalar triplet, χ, with hypercharge 1. Then, $1/\Lambda \approx \mu/m_\chi^2$ with μ the trilinear coupling of the triplet with the Higgs boson doublets. However, this is not the only possibility. In fact, Eq. (1) can be identically rewritten, after a $SU(2)$ Fierz transformation, as

$$\mathcal{L}_{\text{seesaw}} = -\frac{1}{2}\frac{1}{\Lambda}(\bar{\tilde{l}}\varphi)F(\tilde{\varphi}^\dagger l), \tag{3}$$

which suggests the exchange of a neutral heavy Majorana fermion; then Λ should be the mass of that fermion. Indeed, the seesaw mechanism described above naturally implements this possibility.

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Given the full generality of this mechanism, which naturally relates the smallness of neutrino masses to the new physics scale, it is quite reasonable to try to fit the different neutrino mass models into this description. However, this is not always possible; on one side it can happen that neutrino masses are generated only by operators with higher dimensions (for a recent analysis of the different possibilities see [7]) and on the other side models in which neutrino masses are generated through radiative corrections are also difficult to fit in the simplest scheme.

In this paper, we want to show that the uniqueness of the effective seesaw mechanism can be relaxed a bit if new fields are allowed at the electroweak scale, in particular in models with two light doublets. This opens the door for a new class of effective seesaw mechanisms in which the light neutrino masses are generated radiatively. This fact will allow us to lower the lepton number breaking scale by several orders of magnitude. In addition, as we will see, this mechanism predicts a very particular form for the neutrino mass matrix which seems well suited for explaining both atmospheric and solar neutrino data. This is not strange since a particular realization of this mechanism is the Zee model [8,9] and its variations, for instance, models with spontaneous symmetry breaking of the lepton number by a doublet [10] or models in which the spontaneous breaking of the lepton number is induced by a hyperchargeless triplet [11,12]. This last class of models has the interesting property that the triplet does not contribute to the invisible Z decay width and that the lepton number breaking VEV could be at the electroweak scale (no bounds from red giant cooling by Majoron emission). Models with hyperchargeless triplets have also been discussed recently [13,14] in connection with the results of last standard-model fits. Variations solving the strong CP problem can be found in [15,16]. All of these models predict a Zee-type neutrino mass matrix (for a comprehensive review of extensions of the Higgs sector of the SM see [17,18]). Recent fits to neutrino data [19–23] suggest some type of bimaximal mixing which can be accommodated naturally in this type of models [24–27]. This observation has boosted again the interest of models with Zee-type neutrino matrix [28–34].

In the same way that a variety of seesaw mechanism models can be described as a single effective operator it would be interesting to see if this class of models and perhaps other types of models with two light doublets can be described at low energies with just a few operators.

We will assume that the low-energy (Fermi scale) theory is just the SM model, with no right-handed neutrinos, supplemented by an additional doublet. We will denote the two doublets as φ_1 and φ_2. Then, in principle, nothing forbids that both doublets couple to the two types of quarks and to the leptons. However, it is well known that this, in general, will lead to neutral current flavor changing problems [35]. Therefore, for the moment we will consider that only one of the doublets, φ_1, couples to the fermions. This can be achieved naturally by assigning an additional conserved charge to the doublet φ_2, lepton number, for example. Of course, this additional charge should be explicitly broken in the Higgs potential in order to avoid the appearance of a Goldstone boson once the doublet φ_2 acquires a VEV (models in which the lepton number is broken spontaneously by a doublet [10,36,37] are excluded by the invisible Z width measured at the CERN e^+e^- collider LEP). Therefore, for the moment we will assume the SM Yukawa couplings to leptons

$$\mathcal{L}_Y = \overline{l} Y e_R \varphi_1 + \text{H.c.} \tag{4}$$

Again, $SU(2)$ and flavor indices have been suppressed and Y is a 3×3 complex matrix in flavor space. However, because no right-handed neutrinos have been introduced in the model and because only φ_1 couples to leptons, it can be chosen, without loss of generality, as diagonal with all its elements real and positive. As in the SM, neutrinos remain massless because there are no right-handed neutrinos and because lepton number is conserved.

Now we will assume that this model is just the low-energy manifestation of a more complete theory which will only show up at higher energies. As discussed above, if the scale of new physics is high enough its effects can be be parametrized by operators with higher dimensionality:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{2\text{HSM}} + \frac{1}{\Lambda}\mathcal{L}_1 + \frac{1}{\Lambda^2}\mathcal{L}_2 + \cdots. \tag{5}$$

Here $\mathcal{L}_{2\text{HSM}}$ represents the renormalizable Lagrangian we just outlined, which is a minimal extension of the SM containing two doublets.

If the theory at the Fermi scale contains two doublets, one can write four independent dimension five operators[1]

$$\mathcal{L}_{T_1} = -\frac{1}{8}\frac{1}{\Lambda}(\bar{\tilde{l}} T_1 \vec{\tau} l)(\tilde{\varphi}_1^\dagger \vec{\tau} \varphi_1),$$

$$\mathcal{L}_{T_2} = -\frac{1}{8}\frac{1}{\Lambda}(\bar{\tilde{l}} T_2 \vec{\tau} l)(\tilde{\varphi}_2^\dagger \vec{\tau} \varphi_2), \tag{6}$$

$$\mathcal{L}_{T} = -\frac{1}{4}\frac{1}{\Lambda}(\bar{\tilde{l}} T \vec{\tau} l)(\tilde{\varphi}_2^\dagger \vec{\tau} \varphi_1), \tag{7}$$

and

$$\mathcal{L}_{S} = -\frac{1}{4}\frac{1}{\Lambda}(\bar{\tilde{l}} S l)(\tilde{\varphi}_2^\dagger \varphi_1). \tag{8}$$

Operators $\mathcal{L}_{T_1}$, $\mathcal{L}_{T_2}$ can be excluded by the same symmetry used to forbid Yukawa couplings of the doublet φ_2 to the fermions. For instance, one can assign lepton numbers to φ_2 in such a way that it does not have Yukawa couplings to fermions while the couplings $\mathcal{L}_T$ and $\mathcal{L}_S$ remain allowed.

[1]In general one also expects higher-dimension operators which could contribute to interesting processes as μ-e conversion in nuclei, $\mu\to e\gamma$, etc. For instance, when a charged scalar is integrated out all those operators appear at one loop [38].

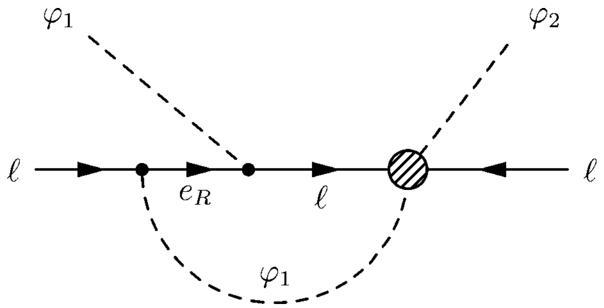

FIG. 1. Diagram contributing to the mixing of the singlet and triplet operators.

$L(\varphi_2)=-2$ will do the job and will forbid operators $\mathcal{L}_{T_1}$, $\mathcal{L}_{T_2}$. Therefore, in the following we will only consider operators $\mathcal{L}_T$ and $\mathcal{L}_S$.

It is important to notice that the operator in Eq. (8) does not exist with only one doublet because the singlet coupling of two scalar doublets is antisymmetric [in $SU(2)$ components it is just $\epsilon_{ij}\varphi_{2i}\varphi_{1j}$]. In addition, one can see that, due to Fermi statistics, the 3×3 matrix in flavor space, T, is symmetric while the singlet coupling S is a complex antisymmetric matrix.

When the Higgs boson fields develop a charge conserving VEV, operator $\mathcal{L}_T$ gives rise to a neutrino Majorana mass. However, the singlet operator $\mathcal{L}_S$ does not give any mass to the neutrinos because the product of the two doublets has hypercharge 1, and a singlet with hypercharge 1 also has charge 1 [different from the triplet combination (7) which has neutral components]. For this reason the singlet operator does not seem very interesting at first sight. However, a very interesting situation arises when, for some reason (limited particle content of the full theory), only operator (8) arises at tree level.[2] As commented before, it cannot give rise to neutrino masses after spontaneous symmetry breaking. However, one expects that renormalization effects will mix all operators with the same dimensionality and the same quantum numbers. Therefore, Eqs. (8) and (7) will mix under the renormalization group and operator (7) will be generated at one loop even if it did not appear at tree level. In fact, one can easily see, by computing the diagram in Fig. 1 (and the crossed diagram) and taking the divergent part, that the matrix T obeys the following renormalization-group equation (RGE):

$$\mu\frac{d}{d\mu}T=\frac{1}{(4\pi)^2}(SYY^\dagger+Y^*Y^TS^T)+\cdots. \tag{9}$$

Here, the dots represent extra contributions to the renormalization group of the T matrix which are proportional to the T matrix itself and, therefore, cannot generate it if it did not exist at some scale. This RGE is peculiar with respect to the RGE we are used to seeing for the Yukawa couplings, in both the SM and the MSSM, in that there is a piece that does not depend on the T matrix. In both the SM and the MSSM there are several chiral symmetries broken only by the Yukawa couplings that ensure that the RGE of those couplings should transform in a covariant way with respect to those symmetries. This ensures, in this type of theory, that fermion masses cannot be generated through radiative corrections. Equation (9) is not of this type and, therefore, even if the coupling T did not exist at the scale Λ at which the operators were generated, it will arise through operator mixing. It is very easy to integrate Eq. (9) by keeping only the leading logarithm [a more sophisticated integration can be performed by taking into account the running of the standard Yukawa couplings, the running of the S coupling itself, and the extra couplings we neglected in Eq. (9), however, this effect is higher order in the couplings and since the couplings are small we expect a small effect]. The result is that

$$T(m_Z)\approx\frac{1}{(4\pi)^2}(SYY^\dagger+Y^*Y^TS^T)\log\left(\frac{m_Z}{\Lambda}\right)+T(\Lambda), \tag{10}$$

where we have identified m_Z with the Fermi scale. We will assume that $T(\Lambda)$ is not generated at tree level. Of course $T(\Lambda)$ could also pick up contributions at one loop (or higher loops). To compute them one would need to know the details of the full theory in which our effective theory is embedded. However, if Λ is large enough, $T(m_Z)$ will be dominated by the model independent logarithmic piece in Eq. (10) which can be computed in the effective theory. So, as a first approximation we will assume $T(\Lambda)\approx 0$ and further on we will keep $T(\Lambda)$ only when the logarithmic pieces vanish.

After spontaneous symmetry breaking, if both φ_1 and φ_2 develop a VEV, operator (7) will give rise to a neutrino mass matrix for the left-handed neutrinos given by

$$M_\nu\approx\tan\beta\frac{1}{(4\pi)^2}(SYY^\dagger+Y^*Y^TS^T)\log\left(\frac{m_Z}{\Lambda}\right)\frac{v_1^2}{\Lambda}, \tag{11}$$

where $v_1=\langle\varphi_1^{(0)}\rangle$, $v_2=\langle\varphi_2^{(0)}\rangle$ are the VEV's of the two doublets and $\tan\beta=v_2/v_1$. The seesaw structure is apparent in the last term. The other factors, however, are also important. First, the neutrino mass matrix comes naturally proportional to the mass of the leptons squared which gives an important suppression since lepton Yukawa couplings are small. Second, it contains the standard loop suppression factor $1/(4\pi)^2$ and, third, the ratio of VEV's of the second doublet v_2 to the standard doublet VEV, $\tan\beta=v_2/v_1$, can give an additional suppression factor. All together, with this mechanism one can achieve the same neutrino masses one would obtain in a standard seesaw mechanism with a Λ which is at least six orders of magnitude smaller. This could put the scale of the new physics responsible for neutrino masses at the reach of the next generation of accelerators if the ratio of VEV's v_2/v_1 and/or the largest S_{ij} are very small. However, per-

[2]An example of this situation is provided by the Zee model in which a charged singlet, h^+, and an extra doublet are introduced with couplings $\bar{\tilde{l}}flh^+$ and $\mu h^-\tilde{\varphi}_2^\dagger\varphi_1$, then, for a heavy h^+ one finds, at tree level, that $S/\Lambda=4\mu f/m_h^2$ while the triplet operator cannot be obtained at tree level.

haps the most interesting aspect of Eq. (11) is the structure of the mass matrix, inherited from the antisymmetric structure of the singlet coupling S; if we choose for the Yukawa coupling of the leptons a diagonal form we find

$$M_\nu \approx -m_0 \begin{pmatrix} 0 & S_{e\mu}(x_e - x_\mu) & S_{e\tau}(x_e-1) \\ S_{e\mu}(x_e-x_\mu) & 0 & S_{\mu\tau}(x_\mu-1) \\ S_{e\tau}(x_e-1) & S_{\mu\tau}(x_\mu-1) & 0 \end{pmatrix}, \tag{12}$$

with $x_e = m_e^2/m_\tau^2$, $x_\mu = m_\mu^2/m_\tau^2$ and

$$m_0 = \tan\beta \frac{m_\tau^2}{\Lambda(4\pi)^2} \log\left(\frac{\Lambda}{m_Z}\right).$$

The structure of this mass matrix is very interesting since all diagonal elements are zero and contains only three free parameters (the three elements of the antisymmetric matrix S). It is convenient to rewrite it as

$$M_\nu \approx \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}. \tag{13}$$

This form of mass matrix has been considered recently in order to fit both atmospheric and solar neutrino data [24–27]. Let us review some of the results; in order to explain solar neutrino data in terms of oscillations one needs a mass-squared difference, Δ_s which is $10^{-11}\ \text{eV}^2 \leqslant \Delta_s \leqslant 10^{-5}\ \text{eV}^2$. A global analysis including the results of SNO suggests mixings close to maximal. On the other hand, in order to explain atmospheric neutrino data one needs a mass squared difference, $\Delta_a \approx 3\times 10^{-3}\ \text{eV}^2$ and also a very large mixing. The particular structure of the obtained mass matrix (it is traceless) implies that the sum of the eigenvalues is zero a fact which constrains the possible solutions. An analysis of the different possibilities in terms of this mass matrix has been carried out in [24–27] where it has been shown that only the case with $m_{e\mu} \sim m_{e\tau}$ and $m_{\mu\tau} \ll m_{e\mu}, m_{e\tau}$ is acceptable. This naturally predicts maximal mixing for solar oscillations which, after SNO, seems to be the only viable possibility. In fact the Zee mass matrix predicts, in this case [28],

$$\sin^2 2\theta_s = 1 - \frac{1}{16}\left(\frac{\Delta_s}{\Delta_a}\right)^2. \tag{14}$$

This is a very strong prediction which is compatible with the low probability, low mass (LOW) and the vacuum (VAC) solutions of the solar neutrino problem. The large mixing angle (LMA) solution, which right now seems to be the preferred solution, would be marginally compatible with this prediction. However, it is important to notice that in general we expect corrections to the Zee mass formula. By requiring that only one doublet couples to fermions we have taken the most restrictive (and predictive) possibility. By relaxing this assumption one can also perfectly fit the LMA region in solar neutrino parameters [29]. But even in the restrictive case in which only one doublet couples to fermions one can also take into account subdominant contributions. In fact, on general grounds we expect modifications to the particular form of the mass matrix with all the elements in the diagonal vanishing. There is no symmetry that enforces this structure, therefore one expects that at some point this structure will receive corrections. This happens, for instance, in the Zee model, where diagonal entries are generated at two loops [31]. For our purposes we can include all those corrections in the initial contributions at the scale Λ, that is, by considering a nonvanishing $T(\Lambda)$ which we could take as a general symmetric matrix. Given the bimaximal mixing required by the data it is natural to parametrize the neutrino mass matrix as follows:

$$M_\nu \approx m_0 \left[\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix} + \epsilon \begin{pmatrix} \gamma_{ee} & \gamma_{e\mu} & \gamma_{e\tau} \\ \gamma_{e\mu} & \gamma_{\mu\mu} & \gamma_{\mu\tau} \\ \gamma_{e\tau} & \gamma_{\mu\tau} & \gamma_{\tau\tau} \end{pmatrix} \right], \tag{15}$$

where, without loss of generality we can take $\gamma_{e\tau} = -\gamma_{e\mu}$ and choose one of the γ's equal to one, defining in this way the normalization of ϵ. We will assume that the first term, generated by running and containing the logarithmic enhancement, is the dominant one and gives the scale of atmospheric neutrinos. The term proportional to ϵ is subdominant and cannot be computed in the effective theory. Its explicit form depends on the details of the underlying theory and it could contain one-loop contributions not enhanced by large logarithms and/or higher loop contributions, depending on the underlying theory. For simplicity, we will also take all the γ's as real. Notice that the mechanism suggested in this paper only gives a leading-order contribution with zeros in the diagonal. The extra structure assumed in the leading mass term ($m_{e\mu} \approx m_{e\tau}$ and $m_{\mu\tau} \approx 0$) must be imposed with some additional symmetries which, however, are not unnatural in this type of model [8]. The modifications introduced by the term proportional to ϵ can be qualitatively very important. One of the most interesting predictions of the leading-order mass matrix is that there is no neutrinoless double beta decay (NDBD); since the NDBD amplitude is proportional to $(M_\nu)_{ee}$ it is obvious that a mass matrix with zeros in the diagonal forbids NDBD. However any correction to the leading-order mass matrix introducing diagonal entries will lead to NDBD. On the other hand, corrections to the leading-order mass matrix are necessary in order to accommodate solar neutrino mass differences and small departures from maximal mixing in solar and/or atmospheric neutrinos. It is important to check how these corrections could modify the sharp predictions of the model.

By diagonalizing the mass matrix at second order in ϵ one easily obtains that

$$\Delta_a \approx m_0^2, \tag{16}$$

$$\sin^2 2\theta_a \approx 1 - \frac{1}{2}[16\gamma_{e\mu} + (\gamma_{\mu\mu} - \gamma_{\tau\tau})^2]\epsilon^2, \tag{17}$$

$$\Delta_s \approx m_0^2(2\gamma_{\mu\tau} + 2\gamma_{ee} + \gamma_{\mu\mu} + \gamma_{\tau\tau})\epsilon, \tag{18}$$

$$\sin^2 2\theta_s \approx 1 - \frac{1}{16}\left(\frac{\Delta_s}{\Delta_a}\right)^2 + \frac{1}{2}[\gamma_{ee}(2\gamma_{\mu\tau} + \gamma_{\mu\mu} + \gamma_{\tau\tau}) - (\gamma_{\mu\mu} - \gamma_{\tau\tau})^2]\epsilon^2. \tag{19}$$

Since Δ_s/Δ_a is known to be small, the natural prediction of the model is just maximal mixing for the two angles to a very good degree of precision. Departures from maximal mixing are allowed naturally for the atmospheric mixing angle since Δ_s does not depend on $\gamma_{e\mu}$ which controls the atmospheric mixing (for $\gamma_{\mu\mu} = \gamma_{\tau\tau}$); therefore it can be made large without any conflict with solar neutrino data. However, to accommodate deviations from maximal mixing in the solar neutrino parameters is more delicate. One needs to make Δ_s small while keeping the last term in Eq. (19) relatively large. This is clearly unnatural. For instance, in the case in which $\gamma_{\mu\mu} = \gamma_{\tau\tau} = 0$ one obtains $\Delta_s/\Delta_a \approx 2(\gamma_{\mu\tau} + \gamma_{ee})$ while $\sin^2 2\theta_s \approx 1 - (\Delta_s/\Delta_a)^2/16 + \gamma_{\mu\tau}\gamma_{ee}$. Therefore, to obtain a very small Δ_s/Δ_a while keeping a sizeable contribution to $\sin^2 2\theta_s$ one would need to fine-tune the couplings in such a way that $\gamma_{ee} \approx -\gamma_{\mu\tau}$. Although it is not natural, this possibility might be interesting because perhaps it is the only way to accommodate LMA within this scheme and because if it is realized in nature it will link the deviation from maximal mixing in solar neutrino oscillations with neutrinoless double beta decay; as commented before, the NDBD amplitude is proportional to the so-called effective neutrino mass $\langle m_\nu \rangle \equiv \Sigma U_{ei}^2 m_{\nu i} = (M_\nu)_{ee}$.

In this work we have shown that, in models that contain in their low-energy (Fermi scale) effective theory two Higgs boson doublets, there are four independent dimension five gauge-invariant operators which violate lepton number. Three of them, which couple leptons to doublets in the triplet channel, generate masses at tree level when the doublets acquire VEV's. The other operator, which couples leptons and doublets in the singlet channel, cannot generate masses at tree level. However, loop corrections mix all operators under the renormalization group and, therefore, the singlet operator also gives rise to neutrino masses when the doublets acquire VEV's.

Under the assumption that only the singlet operator is generated at tree level, at the scale of new physics, one can compute the induced neutrino masses at one loop. Neutrino masses are suppressed by several factors, the loop factor, the masses of charged leptons, and the ratio of the two doublets VEV's, therefore allowing for a new physics scale several orders of magnitude lower than the one needed in tree-level mechanisms for neutrino masses.

Because of the structure of the singlet effective operator, which necessitates antisymmetric couplings in flavor, the obtained mass has the structure of the Zee mass matrix with zero entries in the diagonal. This structure naturally accommodates bimaximal mixing and is well suited for explaining both solar and atmospheric neutrino data. However, in the simplest scheme it is very difficult to accommodate the LMA solution for solar neutrinos because it gives a sharp prediction for $\sin^2 2\theta_s = 1$ once one takes into account the mass differences needed for solar neutrinos.

We have investigated the possibility of accommodating $\sin^2 2\theta_s \neq 1$ in this scheme by considering nonleading contributions to the mass matrix since, on general grounds, one expects to generate nonzero entries for the diagonal elements of the mass matrix. We found that the prediction $\sin^2 2\theta_s = 1$ is quite stable and, only by fine-tuning the parameters of the mass matrix, is it possible to accommodate at the same time $\sin^2 2\theta_s \neq 1$ and $\Delta_s/\Delta_a \ll 1$. This fine-tuning, even though unnatural, has the interesting property that relates the rate of neutrinoless double beta decay to the departure from maximal mixing in solar neutrino oscillations.

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